

Give complete solutions to the following problems. Be sure to provide all the necessary steps to support your answers.

1. Determine if w is in $\text{Nul}(A)$, where

$$w = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}, A = \begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}$$

2. Either use an appropriate theorem to show that the given set, W , is a vector space, or find a specific example to the contrary.

$$\left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a + b + c = 2 \right\}$$

3. Find a nonzero vector in $\text{Nul}(A)$ and a nonzero vector in $\text{Col}(A)$.

$$A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 0 & -5 \\ 0 & -5 & 7 \\ -5 & 7 & -2 \end{bmatrix}$$

4. Determine whether w is in the column space of A , the null space of A , or both, where

$$w = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, A = \begin{bmatrix} -8 & 5 & -2 & 0 \\ -5 & 2 & 1 & -2 \\ 10 & -8 & 6 & -3 \\ 3 & -2 & 1 & 0 \end{bmatrix}$$

5. Find an explicit description of $\text{Nul}(A)$ by listing vectors that span the null space.

$$A = \begin{bmatrix} 1 & 3 & 5 & 0 \\ 0 & 1 & 4 & -2 \end{bmatrix}$$

6. Find A such that the given set is $\text{Col}(A)$.

$$\left\{ \begin{bmatrix} 2s + 3t \\ r - s - 2t \\ 4r + 2 \\ 3r - s - t \end{bmatrix} : r, s \text{ and } t \text{ are real} \right\}$$